

Bose-Einstein condensate in a harmonic trap with an eccentric dimple potential

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Abstract

We investigate Bose-Einstein condensation of noninteracting gases in a harmonic trap with an off-center dimple potential. We specifically consider the case of a tight and deep dimple potential which is modelled by a point interaction. This point interaction is represented by a Dirac delta function. The atomic density, chemical potential, critical temperature and condensate fraction, the role of the relative depth and the position of the dimple potential are analyzed by performing numerical calculations.

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I. INTRODUCTION

The phase space density of a Bose-Einstein condensate (BEC) can be increased by modification of the shape of the potential [1]. “Dimple”-type potentials are the most commonly used potentials for this purpose [2, 3, 4]. A small dimple potential at the equilibrium point of the harmonic trapping potential is used to enhance phase-space density by an arbitrary factor [2]. A tight dimple potential is used for a recent demonstration of caesium BEC a [3]. Quite recently, such potentials were proposed for efficient loading and fast evaporative cooling to produce large BECs [5]. Attractive applications, such as controlling interaction between dark solitons and sound [6], introducing defects such as atomic quantum dots in optical lattices [7], or quantum tweezers for atoms [8] are offered by using tight dimple potentials for (quasi) one-dimensional BECs. Such systems can also be used for spatially selective loading of optical lattices [9]. In combination with the condensates on atom chips, tight and deep dimple potentials can lead to rich novel dynamics for potential applications in atom lasers, atom interferometers and in quantum computations (see Ref. [10] and references therein).

In this paper we continue the discussion of our recent paper [11]. In that paper, we modelled the dimple type potentials by Dirac δ functions and investigated the change of chemical potential, critical temperature and condensate fraction of a harmonic trap with respect to the various strengths of Dirac δ functions. In this paper, we investigate the behavior of the same physical quantities for a δ function which can be located at different positions than the center of a harmonic trap. We find that while a centrally positioned dimple potential is most effective in large condensate formation at enhanced temperatures, there is a critical location for which the condensate fraction and the critical temperature can also be enhanced relatively. This might be useful in spatial fragmentation of atomic condensates.

The paper is organized as follows. In Sect. II, we review shortly the analytical solutions of the Schrödinger equation for a harmonic potential with a finite number of Dirac δ -decorated harmonic potential and give the eigenvalue equation of the harmonic potential with a Dirac δ function. In Sect. III, determining the eigenvalues numerically, we show the effect of the dimple potential on the condensate fraction and the transition temperature and investigate the change of this values with respect to the position of the Dirac δ function. Finally, we

conclude in Sect. IV.

II. HARMONIC POTENTIAL DECORATED WITH DIRAC DELTA FUNCTIONS

We begin our discussion by reviewing the one dimensional harmonic potential decorated with the Dirac δ functions [11]-[14] for the seek of completeness. The potential for is given as:

$$V(x) = \frac{1}{2}m\omega^2 x^2 - \frac{\hbar^2}{2m} \sum_i^P \sigma_i \delta(x - x_i), \quad (1)$$

where ω is the frequency of the harmonic trap, P is a finite integer and σ_i 's are the strengths (depths) of the dimple potentials located at x_i 's with $x_1 < x_2 < \dots < x_P$ with $x_i \in (-\infty, \infty)$. The factor $\hbar^2/2m$ is used for calculational convenience. Negative σ_i value represents repulsive interaction while positive σ_i value represents attractive interaction. The time-independent Schrödinger equation equation for this potential is written as:

$$-\frac{\hbar^2}{2m} \frac{d^2\Psi(x)}{dx^2} + V(x)\Psi(x) = E\Psi(x). \quad (2)$$

By defining $E = (\xi + \frac{1}{2})\hbar\omega$, with ξ a real number, and introducing dimensionless quantities $z = x/x_0$, and $z_i = x_i/x_0$ with $x_0 = \sqrt{\hbar/2m\omega}$, the natural length scale of the harmonic trap, we can re-express Eq. (2) as

$$\frac{d^2\Psi(z)}{dz^2} + \left[\xi + \frac{1}{2} - \frac{z^2}{4} + \sum_i^P \Lambda_i \delta(z - z_i) \right] \Psi(z) = 0, \quad (3)$$

where $\Lambda_i = x_0\sigma_i$. By using transfer matrix approach [11, 14, 16], we get the following eigenvalue equation:

$$1 - \frac{\Lambda_1 D_\xi(z_1) D_\xi(-z_1)}{W} = 0 \quad (4)$$

for ξ and using $E = (\xi + \frac{1}{2})\hbar\omega$. Here, $D_\xi(z)$ and $D_\xi(-z)$ are parabolic cylinder functions and $z_1 = x_1/x_0$. The Wronskian W of $D_\xi(z)$ and $D_\xi(-z)$ is

$$W = W[D_\xi(z), D_\xi(-z)] = \frac{2^{(\xi+3/2)}\pi}{\Gamma(-\frac{\xi}{2}) \Gamma(\frac{1-\xi}{2})} \quad (5)$$

For $z_1 = 0$, these results reduce to the results in Ref. [11, 13, 14].

III. BEC IN A ONE-DIMENSIONAL HARMONIC POTENTIAL WITH A DIRAC δ FUNCTION

In this section we calculate the condensate fraction, chemical potential, critical temperature and density profile for different depths, sizes and positions of a dimple potential modelled by a Dirac δ function. In order to describe the depth and size of a dimple potential in a systematic way we define a dimensionless variable in terms of the strength of the Dirac δ functions as:

$$\Lambda = \sigma \sqrt{\frac{\hbar}{2m\omega}}. \quad (6)$$

We will present our results with respect to Λ and z_1 defined in the previous section.

In ref. [11], we have estimated σ values approximately according the parameters in refs. [17] and [3]. We find that, if $10^8 \text{ 1/m} \leq \sigma \leq 10^{10} \text{ 1/m}$ then $320 \leq \Lambda \leq 32000$ for the experimental parameters $m = 23 \text{ amu}$ (^{23}Na), $\omega = 2\pi \times 21 \text{ Hz}$ [17] and for the experimental parameters $m = 133 \text{ amu}$ (^{133}Cs), $\omega = 2\pi \times 14 \text{ Hz}$ [3]. In ref. [11], we have shown that, even for small Λ values, condensate fraction and critical temperature change considerably. How sensitive such a change would occur depending on the location of the dimple trap was a question left unanswered in Ref.[11].

We begin our discussion by investigating the change of the critical temperature with respect to the position of Dirac δ function. The critical temperature (T_c) is obtained by taking the chemical potential equal to the ground state energy ($\mu = E_g = E_0$) and by solving

$$N \approx \sum_{i=1}^{\infty} \frac{1}{e^{\beta_c \varepsilon_i} - 1}, \quad (7)$$

for β_c where $\beta_c = 1/(k_B T_c)$. For finite N value, we define T_c^0 as the solution of Eq. (7) for $\Lambda = 0$ (only the harmonic trap).

In Eq. (7), ε_i 's are the eigenvalues for the harmonic potential decorated with a single eccentric dimple potential at z_1 . The energies of the decorated states are found by solving Eq. (4) numerically. Then, these values are substituted into the Eq. (7); and finally this equation is solved numerically to find T_c . We obtain T_c for different z_1 values and $\Lambda = 32$ and show our results in Fig. (1). Since harmonic potential is symmetric negative z_1 values will give a the same values for critical temperature with positive ones. As z_1 increases, the energy of the ground state increases so that the critical temperature decreases as z_1

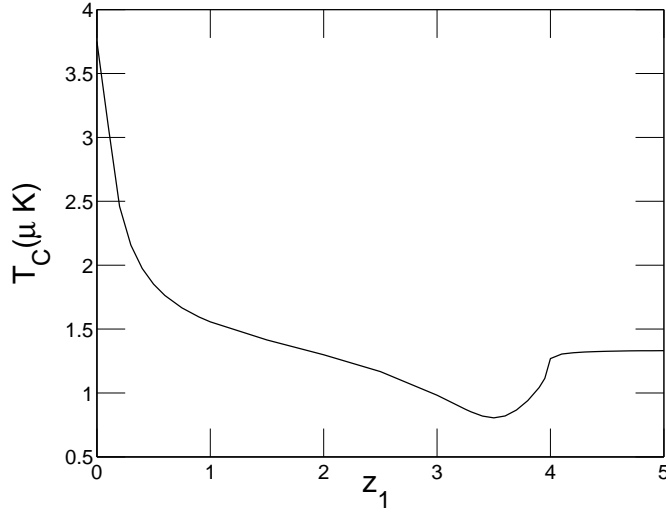


FIG. 1: The critical temperature T_c for $N = 10^4$. Λ is a dimensionless variable defined in Eq. (6). Here we use $m = 23$ amu (^{23}Na) and $\omega = 2\pi \times 21$ Hz [17].

gets larger. On the other hand, as the dimple trap becomes farther to the center of the harmonic trap, the critical temperature cease to decrease and starts rising again as seen in Fig. (1) around $z_1 = 3.5$. Finally, at very large separations between the dimple trap and the harmonic trap center, the critical temperature no longer changes with the location of the dimple trap and saturates at the value corresponding to the of the critical temperature for the single harmonic trap per se. The increase of the critical temperature around $z_1 = 3.5$ can be explained as follows: As z_1 increases it becomes closer to the node of the first excited state wave function of the harmonic potential. At that value the change of the energy eigenvalue of the first excited state vanishes and the difference between the first excited state and ground state eigenvalues increase. Thus, the particles favor the ground state which increases the critical temperature. In Fig. (1) we take $N = 10^4$ and use typical experimental parameters $m = 23$ amu (^{23}Na) and $\omega = 2\pi \times 21$ Hz [17].

For a gas of N identical bosons, the chemical potential μ is obtained by solving

$$N = \sum_{i=0}^{\infty} \frac{1}{e^{\beta(\varepsilon_i - \mu)} - 1} = N_0 + \sum_{i=1}^{\infty} \frac{1}{e^{\beta(\varepsilon_i - \mu)} - 1}, \quad (8)$$

at constant temperature and for given N , where ε_i is the energy of state i . We present the change of μ as a function of T/T_c^0 in Fig.(2) for $N = 10^4$; $\Lambda = 0$; $\Lambda = 32$, $z_1 = 0$ and $\Lambda = 32$ $z_1 = 1$.

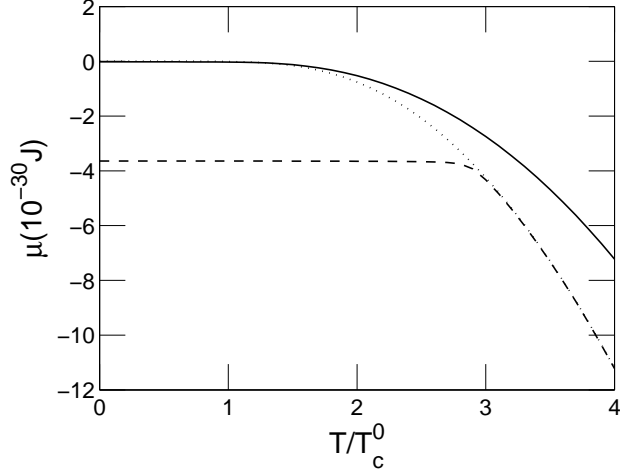


FIG. 2: The chemical potential vs temperature T/T_c^0 for $N = 10^4$. The solid line shows μ only for the harmonic trap. The dotted line shows μ for $z_1 = 1$ and $\Lambda = 32$. The dashed line shows μ for $z_1 = 0$ and $\Lambda = 32$. The other parameters are the same as Fig. 1.

By inserting μ values into the equation

$$N_0 = \frac{1}{e^{\beta(\varepsilon_0 - \mu)} - 1}, \quad (9)$$

we find the average number of particle in the ground state. N_0/N versus T/T_c^0 for $N = 10^4$ $\Lambda = 32$ are shown in Fig. (3). In this figure, the solid line shows the condensate fraction for $z_1 = 0$ and the dashed line shows the condensate fraction for $z_1 = 1$. In Ref. [18], Ketterle et. al mentions that the phase transitions due to discontinuity in an observable macro parameter occurs only in thermodynamic limit, where $N \rightarrow \infty$. However, we make our calculations for a realistic system with a finite number of particles in a confining potential. Thus, N_0/N is a finite non-zero quantity for $T < T_c$ without having any discontinuity at $T = T_c$.

We also investigate the behavior of the condensate fraction as a function of the position of Dirac δ for $\Lambda = 32$, $T = T_c^0$ and present the results in Fig. (4).

Finally, we compare density profiles of condensates for a harmonic trap and a harmonic trap decorated with a delta function ($\Lambda = 3.2$ and $z_1 = 1$) in Fig. (5). Since the ground state wave functions can be calculated analytically for both cases, we find the density profiles by taking the absolute square of the ground state wave functions. Comparing the graphics of density profiles, we see that an offcenter dimple potential maintain a higher density at the position of the Dirac δ function which may be utilized for the fragmentation of a BEC.

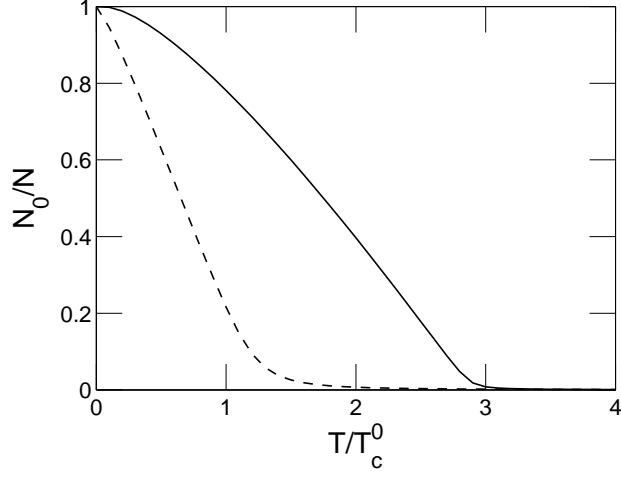


FIG. 3: N_0/N vs T/T_c^0 for $N = 10^4$ and $\Lambda = 32$. The solid line for $z_1 = 0$, the dashed line for $z_1 = 1$. The other parameters are the same as Fig. 1.

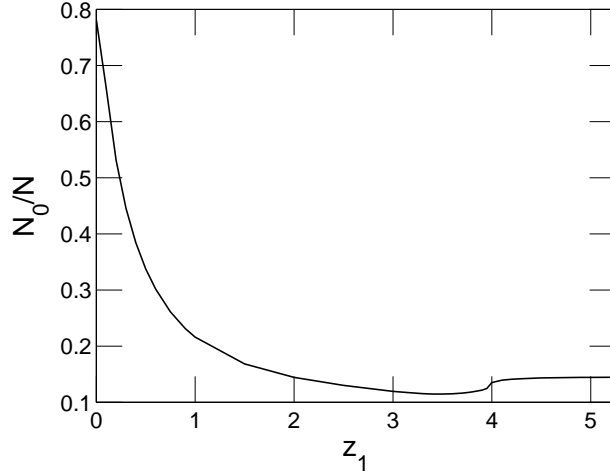


FIG. 4: N_0/N vs T/T_c^0 for $N = 10^4$ and $\Lambda = 32$. The other parameters are the same as Fig. 1.

IV. CONCLUSION

We have investigated the effect of the location of the tight dimple potential on the results reported recently in our paper [11]. We model the tight dimple potential with the Dirac δ function. This allows for analytical expressions for the eigenfunctions of the system and simple eigenvalue equations greatly simplifies subsequent numerical treatment. We have

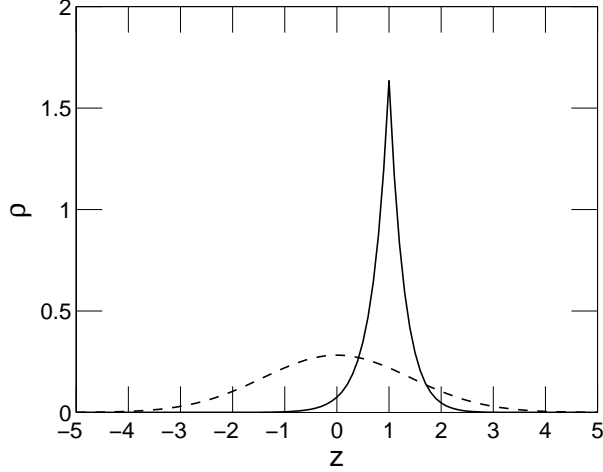


FIG. 5: Comparison of density profiles of a BEC in a harmonic trap with a BEC in a harmonic trap decorated with a δ function ($\Lambda = 3.6$, $z_1 = 1$). The solid curve is the density profile of the BEC in decorated potential. The dashed curve is the density profile of the 1D harmonic trap ($\Lambda = 0$). The parameter z is dimensionless length defined after Eq. (2). The other parameters are the same as Fig. 1.

calculated the critical temperature, chemical potential, condensate fraction and presented the effects of the location of the dimple potential. We find that the dimple type potentials are most effective when they are applied to the center. While it is also advantageous to place the dimple potential at the nodes of the excited state, where our results revealed a relative enhancement of the critical temperature and the condensate fraction. Determining the density profiles of the BECs in the harmonic trap and in the decorated trap with the Dirac δ function at this critical position, we argued that eccentric dimple trap at such a critical location can be used for spatial fragmentation of large, enhanced BECs.

The presented results are obtained for the case of noninteracting and one dimensional condensates for simplicity. In such a case, stability of the condensate may become questionable and should be addressed separately in detail. [19]. The treatment should be extended for the case of interacting condensates in larger (or quasi) dimensional traps in order to make the results more relevant to experimental investigations.

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Figure Captions

Fig. 1 The critical temperature T_c for $N = 10^4$. Λ is a dimensionless variable defined in Eq. (6). Here we use $m = 23$ amu (^{23}Na) and $\omega = 2\pi \times 21$ Hz [17].

Fig. 2 The chemical potential vs temperature T/T_c^0 for $N = 10^4$. The solid line shows μ only for the harmonic trap. The dotted line shows μ for $z_1 = 1$ and $\Lambda = 32$. The dashed line shows μ for $z_1 = 0$ and $\Lambda = 32$. The other parameters are the same as Fig. 1.

Fig. 3 N_0/N vs T/T_c^0 for $N = 10^4$ and $\Lambda = 32$. The solid line for $z_1 = 0$, the dashed line for $z_1 = 1$. The other parameters are the same as Fig. 1.

Fig. 4 N_0/N vs T/T_c^0 for $N = 10^4$ and $\Lambda = 32$. The other parameters are the same as Fig. 1.

Fig. 5 Comparison of density profiles of a BEC in a harmonic trap with a BEC in a harmonic trap decorated with a δ function ($\Lambda = 3.6$, $z_1 = 1$). The solid curve is the density profile of the BEC in decorated potential. The dashed curve is the density profile of the 1D harmonic trap ($\Lambda = 0$). The parameter z is dimensionless length defined after Eq. (2). The other parameters are the same as Fig. 1.

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